

$ee^+ \rightarrow \pi^0 \gamma$ and form factor of $\pi^0 \gamma^* \gamma^*$

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Abstract

The form factor $\pi^0 \gamma^* \gamma^*$ is obtained to the next leading order of derivative expansion of the chiral anomaly and the VMD. As a test the form factor $\pi^0 \gamma^* \gamma$ has been used to calculate the cross section of $ee^+ \rightarrow \pi^0 \gamma$. Theory agrees with data well.

A measurement of the muon g-2 with high accuracy has been reported[1] as

$$a_\mu = 11659202(14)(6) \times 10^{-10}.$$

The hadronic contributions to the muon g-2 consist of vacuum polarization, high order corrections, and light-by-light scattering. The contribution of light-by-light scattering to the muon g-2(a^{lbl}) has been studied[2,3]. The sign problem no longer exists. The pseudoscalar poles play a dominant roles, especially, the pion pole. Various transition form factor of $\pi^0\gamma^*\gamma^*$ have been used in the calculation of a^{lbl} . The Wess-Zumino-Witten term, Vector Meson Dominance, and ENJL model are all used to obtain the transition form factor of $\pi^0\gamma^*\gamma^*$. In Ref.[3] four different form factors of $\pi^0\gamma^*\gamma^*$ are used to calculate the contribution to a^{lbl} .

The measurements of the form factor $\pi^0\gamma\gamma^*$ with one photon on mass shell have lasted for a long time[4,5]. In the timelike region the slope of the form factor of $\pi^0 \rightarrow \gamma e^+ e^-$

$$F(q^2) = 1 + a \frac{m_{e^+e^-}^2}{m_{\pi^0}^2}$$

has been measured[5]. The measured value of the slope falls in a wide range of -0.24 to 0.12. CELLO and CLEO[4] have measured the form factors of $P\gamma\gamma^*$ in the range of large q^2 . The PrimEx Coll. of JLab is going to do direct precision measurements of the form factor of $\pi^0\gamma\gamma^*$ at small values of q^2 , $0.001GeV^2 \leq q^2 \leq 0.5GeV^2$ [6]. On the other hand, the form factor of $\pi^0\gamma\gamma^*$ has been studied by various theoretical approaches[7].

The form factor of $\pi^0\gamma^*\gamma^*$ is related to the chiral anomaly. When two photons are on mass shell in the chiral limit the vertex $\pi^0\gamma\gamma$ is the Adler-Bell-Jackiw anomaly(ABJ)[8] and the amplitude is

$$A_\pi = \frac{2\alpha}{\pi f_\pi}. \quad (1)$$

In the chiral limit the form factor $\pi^0\gamma^*\gamma^*$ is via the Vector Meson Dominance(VMD) derived from the anomalous vertex $\pi\omega\rho$. This vertex is obtained from the Bardeen form of the Wess-Zumino-Witten(WZW) anomalous Lagrangian of pseudoscalar, vector, and axial-vector fields, which has been presented by Kaymakalan, Rajeev, and Schechter (KRS)[9]

$$\begin{aligned} \mathcal{L} = & \frac{N_c}{(4\pi)^2} \frac{2}{3} \varepsilon^{\mu\nu\alpha\beta} \omega_\mu \text{Tr} \partial_\nu U U^\dagger \partial_\alpha U U^\dagger \partial_\beta U U^\dagger \\ & + \frac{2N_c}{(4\pi)^2} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \text{Tr} \{ i [\partial_\beta U U^\dagger (\rho_\alpha + a_\alpha) - \partial_\beta U^\dagger U (\rho_\alpha - a_\alpha)] \\ & - (\rho_\alpha + a_\alpha) U (\rho_\beta - a_\beta) U^\dagger - 2\rho_\alpha a_\beta \}, \end{aligned} \quad (2)$$

$$\mathcal{L}_{\pi\omega\rho} = -\frac{3}{\pi^2 g^2 f_\pi} \pi_i \varepsilon^{\mu\nu\lambda\beta} \partial_\mu \rho_\nu^i \partial_\lambda \omega_\beta, \quad (3)$$

where g is the universal coupling constant which is introduced by normalizing the vector fields

$$\rho_\mu^i \rightarrow \frac{1}{g} \rho_\mu^i, \quad \omega_\mu \rightarrow \frac{1}{g} \omega_\mu.$$

The universal coupling constant g appears in the VMD[10]

$$\frac{1}{2} eg \left\{ -\frac{1}{2} F^{\mu\nu} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) + A^\mu j_\mu \right\}$$

$$\frac{1}{6}eg\{-\frac{1}{2}F^{\mu\nu}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu) + A^\mu j_\mu^0\}. \quad (4)$$

The ABJ anomaly is derived from Eqs.(3,4).

The processes contributing to the form factor $\pi^0\gamma^*\gamma^*$ are shown in Fig.1(a-d) and the matrix element is expressed as

$$\langle \gamma_1\gamma_2 | S | \pi^0 \rangle = -i(2\pi)^4 \delta^4(p - q_1 - q_2) \frac{1}{\sqrt{8m_\pi\omega_1\omega_2}} \varepsilon^{\mu\nu\lambda\beta} \epsilon_\mu(1) \epsilon_\nu(2) q_{1\lambda} q_{2\beta} \frac{2\alpha}{\pi f_\pi} F(q_1^2, q_2^2). \quad (5)$$

Using Eqs.(3,4), the vertices in Fig.1 are obtained and the form factor is determined

$$F(q_1^2, q_2^2) = \frac{1}{2} \left\{ \frac{m_\rho^2 m_\omega^2}{(q_1^2 - m_\rho^2)(q_2^2 - m_\omega^2)} + \frac{m_\rho^2 m_\omega^2}{(q_1^2 - m_\omega^2)(q_2^2 - m_\rho^2)} \right\}, \quad (6)$$

where q_1, q_2 , and p are momentum of two photons and pion respectively. Eq.(6) shows that the form factor is dominated by the poles of vector mesons. This form factor is determined by the Bardeen form of the WZW anomaly presented by KRS and the VMD.

It is necessary to point out that in effective chiral theory of mesons derivative expansion is taken as the low energy approximation. The chiral anomalous Lagrangian(2) is at the 4th order(the lowest order) of covariant derivative expansion. According to Eq.(6), the slope of the form factor is determined by the masses of the vector mesons only. In principle, the chiral anomaly should include terms at higher orders in covariant derivatives. Of course, in the chiral limit the terms at higher orders don't contribute to $\pi^0\gamma\gamma$ (two photons are on mass shell). However, the terms at higher order contribute to $\mathcal{L}_{\pi\omega\rho}$, therefore, they contribute

to $F(q_1^2, q_2^2)$. We use the form factor of charged pion to illustrate the correction of the ρ pole-form factor. It is known that the ρ pole-form factor of charged pion has problems: the radius is less than data by about 10%, in the space-like region the form factor decreases too fast and in the time-like region it decreases too slow[11]. In Ref.[12] by calculating the next leading order in derivative expansion besides the ρ pole an intrinsic form factor

$$f_{\rho\pi\pi}(q^2) = 1 + \frac{q^2}{2\pi^2 f_\pi^2} \left\{ \left(1 - \frac{2c}{g}\right)^2 - 4\pi^2 c^2 \right\} \quad (7)$$

has been found, where

$$c = \frac{f_\pi^2}{2gm_\rho^2}.$$

The intrinsic form factor $f_{\rho\pi\pi}$ remedies these problems. The form factor $F_\pi(q^2)$ agrees with data in both space-like and time-like regions($q^2 \sim 1.4GeV^2$). On the other hand, the radius of charged pion obtained from ρ pole is

$$\langle r^2 \rangle_\pi = 0.395 fm^2$$

and with the contribution of the additional form factor(7) we obtain

$$\langle r^2 \rangle_\pi = \frac{6}{m_\rho^2} + \frac{3}{\pi^2 f_\pi^2} \left\{ \left(1 - \frac{2c}{g}\right)^2 - 4\pi^2 c^2 \right\} = 0.452 fm^2.$$

The data is $(0.44 \pm 0.03) fm^2$ [13]. 13% of the radius comes from the intrinsic form factor $f_{\rho\pi\pi}$. The contribution of the next leading order to the slope is at the same order as the one from the ρ -pole.

The effects of next leading order of derivative expansion on the form factor $\pi^0\gamma^*\gamma^*$ needs to be studied.

We have proposed an effective chiral theory of large N_C QCD of pseudoscalar, vector, and axial-vector mesons[14]. In the limit $m_q \rightarrow 0$, this theory is chiral symmetric and has dynamical chiral symmetry breaking. Based on the current algebra the Lagrangian is constructed as

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(x)(i\gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a\gamma_5 + eQ\gamma \cdot A - mu(x))\psi(x) - \bar{\psi}(x)M\psi(x) \\ & + \frac{1}{2}m_0^2(\rho_i^\mu \rho_{\mu i} + \omega^\mu \omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu) \end{aligned} \quad (8)$$

where M is the quark mass matrix

$$\begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix},$$

$v_\mu = \tau_i \rho_\mu^i + \omega_\mu$, $a_\mu = \tau_i a_\mu^i + f_\mu$, $u = \exp\{i\gamma_5(\tau_i \pi_i + \eta)\}$, and m is the constituent quark mass which is related to dynamical chiral symmetry breaking. The kinetic terms of mesons are generated by quark loops. By integrating out the quark fields, the Lagrangian of mesons is obtained. As shown in Ref.[14] the effective Lagrangian of mesons has real and imaginary two parts. All the vertices of chiral anomaly are from the imaginary part and the normal vertices are from the real part. Like all other effective meson theories, the derivative expansion is taken as a low energy approximation. At the leading order(4^{th}) of derivative expansion the

imaginary part of the Lagrangian obtained from Eq.(8) is exact the same as the Bardeen form of the WZW anomaly presented By KRS(2)(see Eqs.(98-99) of Ref.[14]). As a matter of fact, the fields in the Bardeen form of the WZW anomaly presented by KRS needs to be normalized to physical meson fields. The normalizations are carried out by the kinetic terms of the fields of the real part of the Lagrangian, for example, the normalizations of ρ and ω fields mentioned above and

$$\pi \rightarrow \frac{2}{f_\pi} \pi.$$

The universal coupling constant g and f_π are defined as

$$\begin{aligned} g^2 &= \frac{8N_C}{3(2\pi)^4} \int \frac{d^4k}{(k^2 + m^2)^2}, \\ f_\pi^2(1 - \frac{2c}{g})^{-1} &= \frac{16m^2N_C}{(2\pi)^4} \int \frac{d^4k}{(k^2 + m^2)^2} = 6m^2g^2. \end{aligned} \quad (9)$$

In the chiral limit g and f_π are two inputs. g is determined to be 0.39 by fitting the decay rate of $\rho \rightarrow ee^+$ and we take $f_\pi = 0.186GeV$. N_C expansion is revealed from this theory. The tree diagrams are at leading order of N_C expansion and loop diagrams of mesons are at higher orders. At low energies this theory goes back to the Chiral Perturbation Theory(ChPT) and the 10 coefficients of ChPT are determined[15]. The VMD is a natural result of this theory. In this theory both the anomalous vertices (2) and the VMD(10) are derived from the same Lagrangian(8). In the chiral limit there are two parameters g and f_π which have been fixed. Many physics processes have been calculated and theory agrees with data well [12,14,16].

The theory is phenomenological successful. The form factor of charged pion mentioned above is the result of this theory.

In this paper we use the effective chiral theory(8) to study the contributions of the terms at the 6th order in derivative expansion to the form factor $\pi^0\gamma^*\gamma^*$ at energies up to about 1 GeV. The vertices of the processes shown in Fig.1 to next leading order in derivative expansion can be obtained from the imaginary part of the Lagrangian(1) by calculating corresponding quark loops. The related vertices to the sixth order in derivatives are found

$$\mathcal{L}_{\pi^0\gamma\gamma} = -\frac{e^2}{4\pi^2 f_\pi} \left\{ 1 + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g} \right)^2 (q_1^2 + q_2^2 + p^2) \right\} \pi^0 \varepsilon^{\mu\nu\lambda\beta} \partial_\mu A_\nu \partial_\lambda A_\beta, \quad (10)$$

$$\mathcal{L}_{\pi^0\rho\gamma} = -\frac{e}{2g\pi^2 f_\pi} \left\{ 1 + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g} \right)^2 (q_1^2 + q_2^2 + p^2) \right\} \pi^0 \varepsilon^{\mu\nu\lambda\beta} \partial_\mu A_\nu \partial_\lambda \rho_\beta, \quad (11)$$

$$\mathcal{L}_{\pi^0\omega\gamma} = -\frac{3e}{2g\pi^2 f_\pi} \left\{ 1 + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g} \right)^2 (q_1^2 + q_2^2 + p^2) \right\} \pi^0 \varepsilon^{\mu\nu\lambda\beta} \partial_\mu A_\nu \partial_\lambda \omega_\beta, \quad (12)$$

$$\mathcal{L}_{\rho\gamma} = -\frac{e}{4} g (\partial_\mu A_\nu - \partial_\nu A_\mu) \left\{ 1 - \frac{1}{10\pi^2 g^2} \frac{\partial^2}{m^2} \right\} (\partial^\mu \rho^{0\nu} - \partial^\nu \rho^{0\mu}), \quad (13)$$

$$\mathcal{L}_{\omega\gamma} = -\frac{e}{12} g (\partial_\mu A_\nu - \partial_\nu A_\mu) \left\{ 1 - \frac{1}{10\pi^2 g^2} \frac{\partial^2}{m^2} \right\} (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu), \quad (14)$$

$$\mathcal{L}_{\pi^0\omega\rho} = -\frac{3}{\pi^2 g^2 f_\pi} \pi^0 \varepsilon^{\mu\nu\lambda\beta} \partial_\mu \rho_\nu^0 \partial_\lambda \omega_\beta \left\{ 1 + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g} \right)^2 (q_1^2 + q_2^2 + p^2) \right\}, \quad (15)$$

where q_1^2 , q_2^2 , and p^2 are momentum of $\rho(\gamma)$, $\omega(\gamma)$, and π^0 respectively. In the chiral limit, if the two photons are on mass shell Eq.(10) goes back to the ABJ anomaly

$$\mathcal{L}_{\pi^0 \rightarrow \gamma\gamma} = -\frac{\alpha}{\pi f_\pi} \varepsilon^{\mu\nu\lambda\beta} \pi^0 \partial_\mu A_\nu \partial_\lambda A_\beta. \quad (16)$$

Up to the 6th order in derivatives, the form factor of $\pi^0\gamma^*\gamma^*(5)$ is determined to be

$$F(q_1^2, q_2^2) = \frac{1}{2} \left\{ \frac{m_\rho^2 m_\omega^2}{(q_1^2 - m_\rho^2)(q_2^2 - m_\omega^2)} + \frac{m_\rho^2 m_\omega^2}{(q_1^2 - m_\omega^2)(q_2^2 - m_\rho^2)} + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g}\right)^2 (q_1^2 + q_2^2 + p^2) \right\}, \quad (17)$$

in space-like region, and q_1^2 , q_2^2 , and p^2 are momentum of two photons and pion respectively.

Comparing with Eq.(6), in Eq.(17) there is a new term which is from the terms at the 6th order in derivatives.

Put one photon on mass shell (π^0 too), in the chiral limit the form factor of $\pi^0\gamma\gamma^*$ is derived from Eq.(17)

$$F_{\pi^0\gamma\gamma^*}(q^2) = \frac{1}{2} \left\{ \frac{m_\rho^2}{m_\rho^2 - q^2} + \frac{m_\omega^2}{m_\omega^2 - q^2} + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g}\right)^2 q^2 \right\}. \quad (18)$$

At very low energies the slope of the form factor is obtained

$$F_{\pi\gamma\gamma^*}(q^2) = 1 + a \frac{q^2}{m_\pi^2}, \quad (19)$$

$$a = \frac{m_\pi^2}{2} \left(\frac{1}{m_\rho^2} + \frac{1}{m_\omega^2} \right) + \frac{m_\pi^2}{2f_\pi^2} g^2 \left(1 - \frac{2c}{g}\right)^2. \quad (20)$$

$$a = 0.0303 + 0.0157 = 0.046. \quad (21)$$

The first number of Eq.(21) is from the poles of vector mesons and the second number is the contribution of the term at the 6th order in derivatives, which is 34% of the slope. The value of the slope is the prediction of this theory and is a test of the form factor(18).

The cross section of $ee^+ \rightarrow \pi^0\gamma$ has been measured[17]. $\sigma(ee^+ \rightarrow \pi^0\gamma)$ is determined by

the time-like form factor $\pi^0\gamma^*\gamma$

$$\sigma(ee^+ \rightarrow \pi^0\gamma) = \frac{\alpha^3}{6\pi^2 f_\pi^2} \left(1 - \frac{m_\pi^2}{q^2}\right)^3 |F(q^2)_{\pi^0\gamma\gamma^*}|^2. \quad (22)$$

The cross section(22) is a test of the form factor $\pi^0\gamma^*\gamma^*$ obtained in this paper. The time-like form factor in Eq.(22) can be obtained from Eq.(18).

$$F_{\pi^0\gamma\gamma^*}(q^2) = \frac{1}{2} \left\{ \frac{-m_\rho^2 + i\sqrt{q^2}\Gamma_\rho(q^2)}{q^2 - m_\rho^2 + i\sqrt{q^2}\Gamma_\rho(q^2)} + \frac{-m_\omega^2 + i\sqrt{q^2}\Gamma_\omega(q^2)}{q^2 - m_\omega^2 + i\sqrt{q^2}\Gamma_\omega} + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g}\right)^2 q^2 \right\}, \quad (23)$$

where $\Gamma_\omega = 8.44 MeV$ is taken. The decay width of the wide resonance of ρ is expressed as[12]

$$\Gamma_\rho(q^2) = \frac{\sqrt{q^2}}{12\pi g^2} f_{\rho\pi\pi}^2(q^2) \left(1 - \frac{4m_K^2}{q^2}\right)^{\frac{3}{2}}. \quad (24)$$

At $q^2 = m_\rho^2$ $\Gamma_\rho = 150 MeV$ which agrees with data very well. If $q^2 > 4m_K^2$ the KK channels are open. However, in the range $q^2 < 1 GeV^2$ the contribution of KK channels to Γ_ρ can be ignored.

In time-like region because of the effects of narrow resonances the $\rho-\omega$ and $\omega-\phi$ mixings have to be taken into account. In the effective chiral theory[14] in the leading order of N_C expansion the masses of ρ and ω are degenerated. Therefore, in the leading order of N_C expansion the $\rho-\omega$ mixing is kinetic[18]

$$\mathcal{L}_{\rho\omega} = \left\{ -\frac{1}{4\pi^2 g^2} \frac{1}{m} (m_d - m_u) + \frac{1}{24} e^2 g^2 \right\} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu u) (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu u). \quad (25)$$

In Ref.[18] Eq.(25) has been used to study $ee^+ \rightarrow \pi^+\pi^-$. Theory agrees with data well. $m_d - m_u$ is determined to be 4.24 ± 0.32 MeV which is in good agreement with the one presented by Leutwyler[19]. In the effective chiral theory[14] nondiagonal element of mass matrix of ω and ϕ cannot be generated. However, kinetic mixing exists. We take the $\omega - \phi$ mixing as

$$\mathcal{L}_{\omega\phi} = \frac{1}{2}st(\partial_\mu\phi_\nu - \partial_\nu\phi_\mu)(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu), \quad (26)$$

where st is a parameter. Taking $\mathcal{L}_{\rho\omega}$ and $\mathcal{L}_{\omega\phi}$ into account, the form factor $\pi^0\gamma^*\gamma$ is expressed as

$$\begin{aligned} F_{\pi^0\gamma\gamma^*}(q^2) = & \frac{1}{2} \left\{ \frac{-m_\rho^2 + i\sqrt{q^2}\Gamma_\rho(q^2)}{q^2 - m_\rho^2 + i\sqrt{q^2}\Gamma_\rho(q^2)} + \frac{-m_\omega^2 + i\sqrt{q^2}\Gamma_\omega(q^2)}{q^2 - m_\omega^2 + i\sqrt{q^2}\Gamma_\omega} + \frac{g^2}{2f_\pi^2} \left(1 - \frac{2c}{g}\right)^2 q^2 \right. \\ & + \sqrt{2}st \frac{q^2}{q^2 - m_\phi^2 + i\sqrt{q^2}\Gamma_\phi} \frac{q^2}{q^2 - m_\omega^2 + i\sqrt{q^2}\Gamma_\omega} \left. \right\} \\ & + \frac{10}{3} \left\{ \frac{1}{4\pi^2 g^2} \frac{1}{m} (m_d - m_u) - \frac{1}{24} e^2 g^2 \right\} \frac{q^4}{(q^2 - m_\rho^2 + i\sqrt{q^2}\Gamma_\rho(q^2))(q^2 - m_\omega^2 + i\sqrt{q^2}\Gamma_\omega)}. \end{aligned} \quad (27)$$

$st = 0.0167$ is chosen to fit the cross section of $ee^+ \rightarrow \pi^0\gamma$ at $q^2 = m_\phi^2$ [20]. The results are shown in Fig.2. Theory agrees with data well.

To conclude, the form factor of $\pi^0\gamma^*\gamma^*$ has been found up to next leading order in derivative expansion. The slope of the form factor is predicted. The cross section of $ee^+ \rightarrow \pi^0\gamma$ has been computed. Theory agrees well with data up to $q^2 \sim 1\text{GeV}^2$.

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- [20] I have used Eq.(26) and the value of s_t to study $ee^+ \rightarrow 3\pi$. Correct result for $\phi \rightarrow 3\pi$ is obtained. The study will be presented somewhere else.

Figure Captions

FIG. 1. Processes contributing to $\pi^0\gamma^*\gamma^*$

FIG. 2. Cross section of $ee^+ \rightarrow \pi^0\gamma$

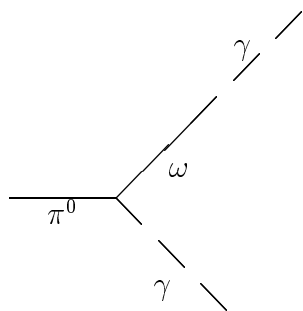


Fig.1(a)

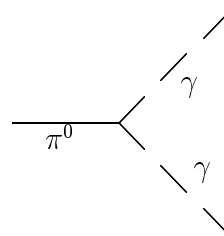


Fig.1(b)

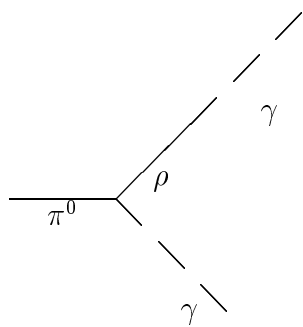


Fig.1(c)

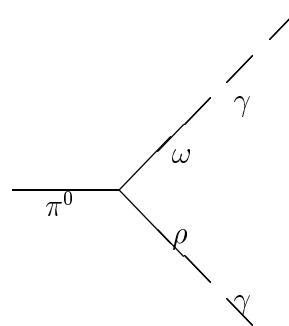


Fig.1(d)

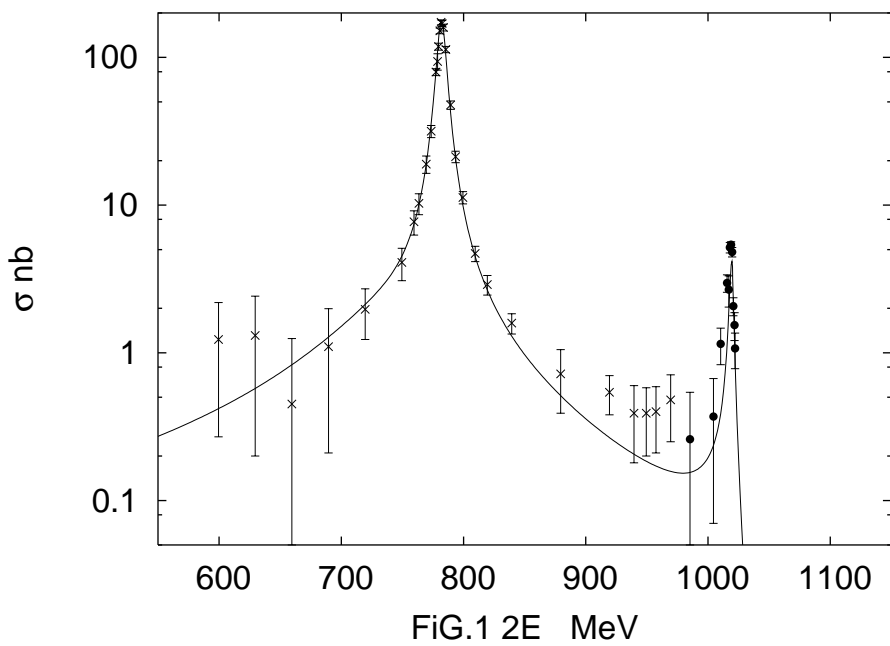


FIG. 2.